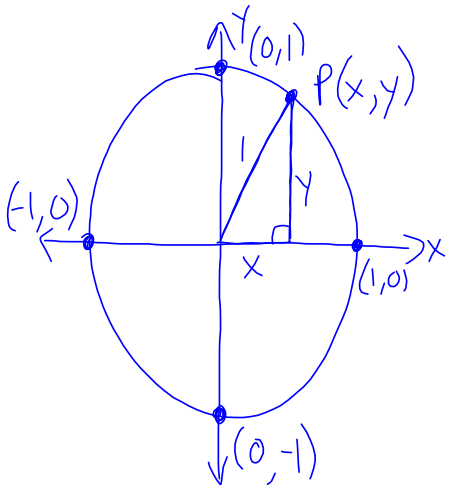
4.2 The Unit Circle

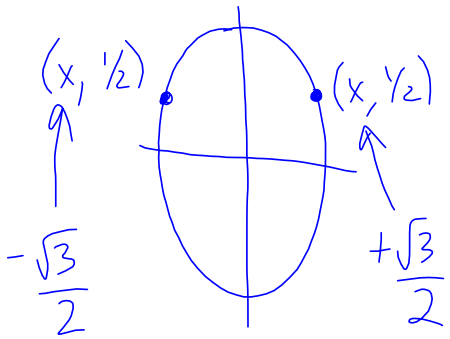


The radius of a unit circle is 1 unit, and the center is at (0,0).

$$x^2 + y^2 = 1$$

The equation of the unit circle.

ex) $(x, \frac{1}{2})$ is a point on the unit circle. Find x.



$$x^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$x^2 + \frac{1}{4} = 1$$

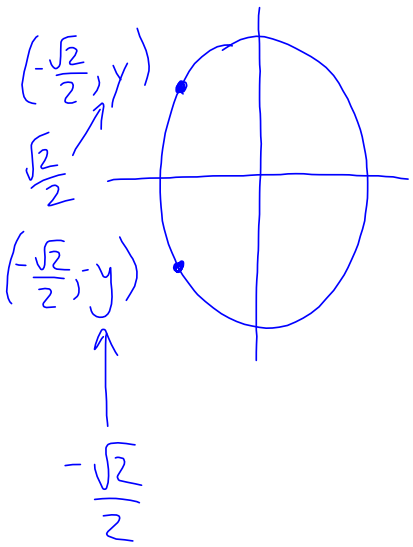
$$x^2 = \frac{3}{4}$$

$$x = \pm\sqrt{\frac{3}{4}}$$

$$x = \pm\frac{\sqrt{3}}{\sqrt{4}}$$

$$x = \pm\frac{\sqrt{3}}{2}$$

ex) $\left(\frac{-\sqrt{2}}{2}, y\right)$ is a point on the U.C. Find y.



$$\left(-\frac{\sqrt{2}}{2}\right)^2 + y^2 = 1$$

$$\frac{2}{4} + y^2 = 1$$

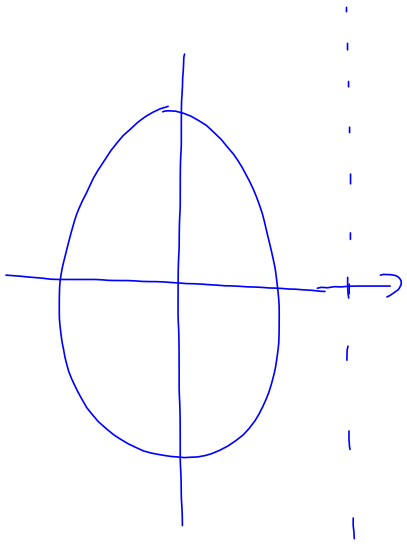
$$y^2 = \frac{1}{2}$$

$$y = \pm\sqrt{\frac{1}{2}}$$

$$y = \pm\frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$$

ex) $(2, y)$ is on the U.C. Find y.

$$2^2 + y^2 = 1$$
$$4 + y^2 = 1$$
$$-4 \quad -4$$
$$y^2 = -3$$
$$y = \pm\sqrt{-3}$$

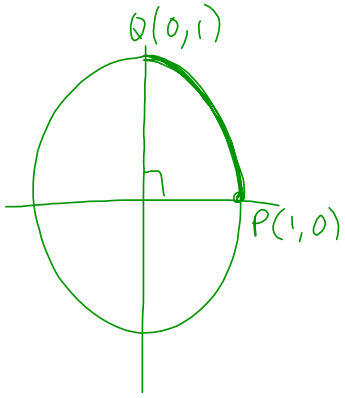


It's not possible since the radius is only 1.

The point $(2, y)$ is on the circumference of a circle centered at $(0,0)$ with radius $= 3$.

$$2^2 + y^2 = 3^2$$
$$4 + y^2 = 9$$
$$y = \pm\sqrt{5}$$

ex) a) Sketch a unit circle and mark P(1,0) and Q(0,1). How long is the arc $\overset{\frown}{PQ}$?



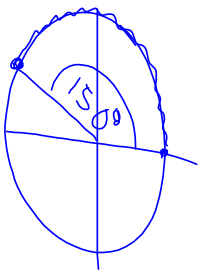
$arc = \theta \times r$

$= \frac{\pi}{2} \times 1$

$= \pi/2$

$90° \times \frac{\pi}{180°} = \frac{\pi}{2}$

b) Sketch an angle in S.P. on a UC of 150°. How long is the arc that subtends the angle?



$arc = \theta \times r$

$= \frac{5\pi}{6} \times 1$

$= \frac{5\pi}{6}$

$150° \times \frac{\pi}{180°} = \frac{5\pi}{6}$

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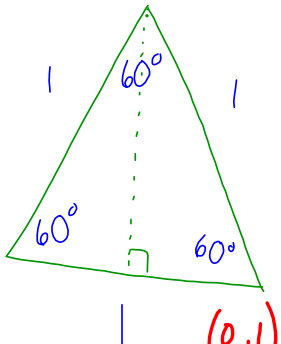
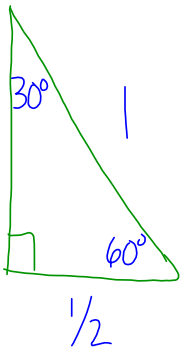
#1-3, 10, C3, C4

The Angles of the Unit Circle



Equilateral triangle w sides = 1.



$a = \frac{\sqrt{3}}{2}$

$a^2 + \left(\frac{1}{2}\right)^2 = 1^2$

$a^2 = 1 - \frac{1}{4}$

$a = \pm\sqrt{3/4} = \pm\frac{\sqrt{3}}{2}$